A THEORY OF DEEP DYNAMIC WING STALL

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Nonsteady flow about an airfoil with a nontrivial rate of change in the angle of attack ($d\alpha/dt = \dot{\alpha}$), occurring during large vibrations of the airfoil, is accompanied by flow separation and phenomenon which can be termed dynamic hysteresis of the airfoil's aerodynamic characteristics (in contrast to the familiar hysteresis occurring at $\dot{\alpha} = 0$). Such separation has been intensively studied in recent years in connection with its importance in the solution of problems in aviation, hydrodynamics, and wind-power engineering. Some of these investigations are discussed in the survey [1], which presents a classification of the different stages of development of dynamic stalling. This classification included the final stage of deep dynamic stall, which is accompanied by flow separation from the leading and trailing edges of the airfoil and is associated with the highest-amplitude dynamic hysteresis of the aerodynamic characteristics. An important aspect of deep dynamic stall is that the aerodynamic loads experienced at the given angle of attack with nontrivial $\dot{\alpha}$ for the airfoil may be greater than the loads experienced at the same angle of attack when $\dot{\alpha} = 0$. This property of deep dynamic stall was firmly established experimentally in 1985-1986 [2-4]. A new phase of study was the transition from the investigation of dynamic stall with periodic (mainly harmonic) oscillations of the angle of attack of the airfoil to the examination of the same phenomenon with aperiodic oscillations. Specifically, attention began to be paid to the case when the angle of attack changes with a constant angular velocity ($\dot{\alpha}$ = const), beginning from zero and proceeding to a large value ($\alpha = 60°$ or $\alpha = 90°$) before becoming constant or slowly decreasing.

However, such increases in the drag and lift coefficients of an airfoil with $\dot{\alpha}$ = const, examined in relation to the static values fixed in [3], are sensational and are quite unexpected for aerodynamics. In fact, the drag coefficient of an airfoil at $\alpha = 90°$ under static conditions – which is close to the coefficient of a flat plate with the same angle of attack ($c_x \simeq 2.0$) – reaches $c_x \simeq 5.0$ at $\omega = \dot{\alpha}b/2U_\infty = 0.5$ (where b is the chord of the airfoil and U_∞ is the velocity of the undisturbed flow) and $c_x > 11.0$ at $\omega \simeq 1.0$ [3]. The lift coefficient of the airfoil undergoes the same type of sensational change. Thus, with $\alpha = 45°$ in the supercritical region of angles of attack, the maximum value $c_{ymax} = 1.2$ under static conditions, $c_{ymax} = 5.5$ at $\omega = 0.5$, and c_{ymax} approaches 10.0 at $\omega \simeq 1.0$ [3].

Is it possible that a manifold (by a factor from 3 to 9) increase in the aerodynamic loads at the same given angle of attack is seen only at large values of α which are rarely encountered in practice? It turns out that this is not entirely the case. With a reduction in the rate of change in the pitching angle by an order of magnitude (at $\omega = 0.088$), c_{xmax} and c_{ymax} remain on the order of 3.0 [3]. Finally, at $\omega = 0.02$ – which corresponds to the case in which the peripheral velocity of the tip of the airfoil when rotated relative to the point at the middle of the chord is only 2% of the free-stream velocity – c_{ymax} is more than twice as great as c_{ymax} for the airfoil under static flow conditions [4].

Of course, the results obtained in [2-4] at Reynolds numbers Re on the order of 10^5 must be substantiated for Re on the order of 10^6-10^7 in order to establish a correspondence with the known aerodynamic characteristics of airfoils beyond the limits of the transitional region of Re. These characteristics include the data obtained in [1]. Despite this, the experimental results reported in [2-4] present a challenge to theoreticians, since they were not predicted by any of the known theories or models of nonsteady flow about an airfoil. We known of only one successful attempt [5] to describe deep dynamic stall by numerically solving the Navier–Stokes equations for the nonsteady turbulent flow of a gas around a vibrating airfoil. In this case, the pattern of hysteresis of the aerodynamic

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characteristics that was reproduced was to a significant extent similar to that seen in [1] with harmonic vibrations of an airfoil in accordance with the law $\alpha = 15° + 10° \sin \dot{\alpha} t$. As regards large supercritical angles of attack α = 45, 60, and 90° — where the increase in the aerodynamic loads during nonsteady flow about the airfoil with $\dot{\alpha}$ = const is particularly substantial — we know of neither numerical nor theoretical results that explain this increase.

The goal of the present investigation is to explain the sensational experimental results obtained in [2-4] with $\dot{\alpha}$ = const and to construct a theory to explain the hysteresis of the aerodynamic characteristics of the profile of a wing during its vibration in the deep dynamic stall regime described in [1]. The starting point is the theory presented in [6, 7] for quasisteady flow about a wing at subcritical angles of attack. In the deep dynamic stall regime, the wing profile is represented by a flat plate in a flow of an incompressible fluid. The flow has separated from the leading and trailing edges, so that the lift forces that normally act on these edges during nonseparated flow are not realized. It is shown that the main physical reason for the increase in aerodynamic loads at $\omega \ll 1$ and $\dot{\alpha}$ = const — and the reason for dynamic hysteresis during periodic vibration of the airfoil in the deep dynamic stall regime — is a change in the geometry (displacement thickness) of the vortex wake behind the airfoil (in the limiting case being examined here, the disappearance of the displacement half-body of the wake at $\dot{\alpha} > 0$). Thus, thanks to the use of an energy-based approach that requires considerably less information on the details of the global flow than is needed when a purely mechanical approach is used, the quasisteady variant of the model of the second dissipative layer and wake proposed in [8] turns out to be an acceptable means of explaining and even quantitatively describing one of the most complex phenomena in aerohydrodynamics — the phenomenon of deep dynamic wing stall.

1. We will examine separated flow about a plate of infinite span moving in a compressible fluid with the translational velocity U_∞. During the plate's motion, the angle of attack α changes with the constant angular velocity $\dot{\alpha}$ relative to a point on the plate located the distance $x_0 = \bar{x}_0 b/2$ from the plate's middle (b is the chord of the plate).

It is readily seen that at $\dot{\alpha} > 0$ the dissipative vortex wake behind the plate does not have the time to form that it does at $\dot{\alpha} = 0$. In the latter case, there is sufficient time for the formation of a displacement half-body having a thickness, at an infinite distance from the plate, which is equal to the momentum thickness.

We will study the limiting case of nonsteady flow about a plate in the case where the displacement thickness of the wake is zero. In the absence of suction forces on the leading and trailing edges of the plate due to separation of the flow in the deep dynamic stall regime, the resulting force can be applied only in the direction normal to the plate. The circulation $\Gamma = 0$, and the lift is due to the drag applied to the plate [7]. If the aerodynamic loads under these conditions are determined only by the drag, then an energy approach can be used. This simplifies the problem considerably.

The work done by the drag XU_∞ is represented as the sum of the amounts of work $X_I U_\infty + X_D U_\infty$ done, respectively, in changing the kinetic energy of the unsteady flow — which is connected with a change in the apparent additional mass (inertial component) — and in overcoming the loss of energy (dissipation) of the jets separating from the leading and trailing edges of the plate in the absence of suction forces on these edges (dissipative component). The initial data to find these components can be obtained by solving the problem of noncirculatory and nonseparated unsteady flow about a plate of infinite span with U_∞ = const and $\dot{\alpha}$ = const. This problem was solved in accordance with a plan that we developed jointly with Sadovskii [9]. The following formulas, obtained in [9], are used for the initial data:

$$c_{xE} = \pi[(\omega/2) \sin 2\alpha + \omega^2 \bar{x}_0 \cos \alpha]; \tag{1.1}$$

$$c_\tau(\pm 1) = (\pi/2)[\sin \alpha - \omega(1/2 - \bar{x}_0)]^2, \tag{1.2}$$

where c_{xE} is the drag coefficient of the plate in a nonseparated unsteady flow; $c_\tau(\pm 1)$ are the coefficients of the suction forces acting on the leading (+1) and trailing (−1) edges of the plate in the nonseparated unsteady flow.

It is natural to assume that

$$c_{xI} = c_{xE}, \tag{1.3}$$

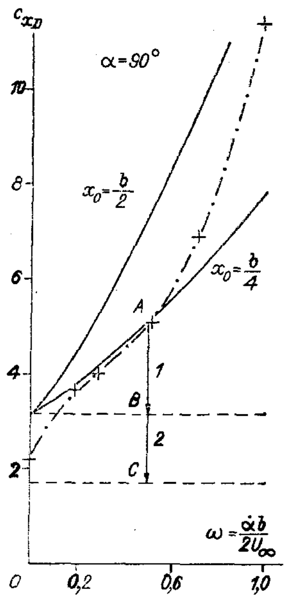

Fig. 1

since in the case of nonseparated flow the work $X_I U_\infty$ ensures a change in the kinetic energy of an unsteady flow about the plate without circulation and without a wake, i.e., under the conditions which correspond in the present limiting case to deep dynamic stall. It is assumed that the local difference in the two flow components near the sharp edges does not violate Eq. (1.3), which connects two integral quantities. However, this local difference does lead to the generation of a dissipative component in the work done by the drag. In the case of steady separated flow about the plate ($\dot{\alpha}$ = 0) with an arbitrary angle of attack α [7], the existence of the dissipative component does lead to the appearance of the dissipative drag coefficient

$$c_{xD} = c_\tau(+1) + c_\tau(-1). \tag{1.4}$$

We will use this relation here to determine the dissipative component of the drag force at $\dot{\alpha} > 0$ on the basis of the above assumption that $\dot{\alpha}$ is large enough so that there is not sufficient time for the formation of a full-sized vortex wake behind the plate. We further assume, however, that $\dot{\alpha}$ is at the same time small enough so that there is sufficient time for the establishment of a steady-state value for the rate of dissipation of the kinetic energy of the jets which separate from the leading and trailing edges. The latter condition is reflected by Eq. (1.4).

The lift of the plate is derived from condition of normality of the resultant relative to the plane of the plate at the given moment of time. The lift is determined by two components of the drag: inertial and dissipative

$$c_y = (c_{xI} + c_{xD})/\mathrm{tg}\alpha. \tag{1.5}$$

Thus, in order to use Eqs. (1.3)-(1.5) to calculate the aerodynamic loads on the wing with $\dot{\alpha}$ = const within the framework of the model of deep dynamic stall being examined here, it turns out to be sufficient to have initial data – the functions $c_{xE} = f(\omega, \bar{x}_0, \alpha)$, $c_\tau(\pm 1) = f(\omega, \bar{x}_0, \alpha)$ [9, Eqs. (1.1) and (1.2)] – for nonseparated noncirculatory unsteady flow about a rotating plate.

2. First we will determine the aerodynamic load in the simplest case, when lift and the inertial component of the drag are absent. It can be seen from Eqs. (1.1), (1.3), and (1.5) that this occurs when $\alpha = 90°$. At $\bar{x}_0 = 1/2$ (when the axis of rotation of the plate is located a distance from the leading edge corresponding to one-fourth of the chord), it follows from Eqs. (1.2), (1.4) that

$$c_{xD} = \pi/2 + (\pi/2)(1 + \omega)^2. \tag{2.1}$$

In Fig. 1, Eq. (2.1) is compared with experimental data [3] (dot–dash line) for an NACA 0015 wing. The data was obtained with Re = 10^5 and values of ω that change within the range 0.088-0.99 at the moment the profile of the wing reaches $\alpha = 90°$. There was no

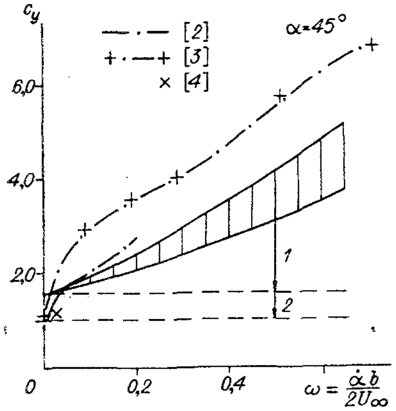

Fig. 2

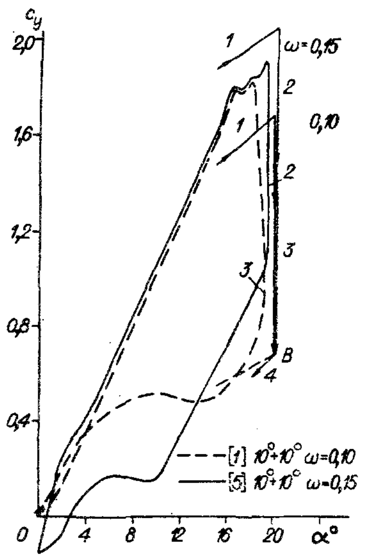

Fig. 3

indication in [3] of the position of the axis of rotation of the airfoil when the experimental data was obtained. To evaluate the effect of the position of the axis of rotation, Fig. 1 presents the theoretical curve at $\bar{x}_0 = 1$ (the axis of rotation coincides with the leading edge of the plate); it can be seen that the position of the rotation axis turns out to have a strong effect on the value of c_{xD}.

The dashed line in Fig. 1 shows the levels corresponding to theoretical values of the drag coefficient of the plate with $\dot{\alpha} = \omega = 0$. These results were obtained from [6] ($\alpha = 90°$) and pertain to the following cases: $c_{xD} \simeq 1.69$, when a fully-formed vortex wake with a displacement thickness $\bar{\delta}^* = c_{xD}/2$ is created behind the plate; $c_{xD} = \pi$, when the displacement thickness of the wake is zero, $\bar{\delta}^* = 0$. The form of the empirical relation reflects the complex nonlinear character of the phenomenon of deep dynamic stall: a rapid increase in c_{xD} at $\omega \ll 1$ which is replaced by a slow increase finally, at $\omega \simeq 1.0$, the attainment of sensationally large values of c_{xD} in excess of 10.0.

The rapid increase in c_{xD} at $\omega \ll 1$ is empirical substantiation for the main idea behind the present investigation: the transition from the value $\omega = 0$ to finite but small values of ω is accompanied by disappearance of the fully formed wake behind the plate and a changeover to the wakeless flow incorporated into our model. At $\omega \to 0$, this flow corresponds to $c_{xD} = \pi$. The slow increase in c_{xD} seen in the tests at $0.1 < \omega < 0.5$ is satisfactorily described by Eq. (2.1), while the more rapid increase in c_{xD} within the range $0.5 < \omega < 1.0$ is within the range of the theoretical estimates – although the latter were obtained with the axis of rotation of the plate in a very advanced position. In any case, the large values of $c_{xD} > 10.0$ [3] lose their sensational aspect, since they can be explained from a theoretical standpoint. In the simple case we are examining – when lift and the inertial component of drag are absent – Fig. 1 shows the possible reduction in the drag coefficient when the rotation of the plate is stopped with $\omega = 0.5$ at $\alpha = 90°$. This information is shown by the arrows (1 corresponds to disappearance of the unsteady component of the drag coefficient while the displacement thickness of the wake remains zero; 2 corresponds to a further decrease in the drag coefficient due to the formation of a full-sized quasisteady vortex wake).

3. Can the present model of deep dynamic stall explain the unexpectedly large maxima of the lift coefficient seen in the American experiments? To answer this question, we compare the theoretical and experimental relations $c_y = f(\omega)$ obtained at $\alpha = 45°$. In neither case do the values of c_y at $\alpha = 45°$ coincide with the values obtained for c_{ymax}. However, the former are close to the latter and can thus serve as a basis of comparison with the relations $c_{xD} = f(\omega)$ at $\alpha = 90°$. These relations also fail to coincide with $c_{xmax} = f(\omega)$ but are close to these functions.

The solid line in Fig. 2 shows the theoretical relation $c_y = f(\omega)$ at $\alpha = 45°$, constructed by means of Eqs. (1.3)-(1.5) with the position of the axis of rotation on the plate

$\bar{x}_0 = 1/2$ (one-fourth of the chord from the leading edge). The vertical dashed line in Fig. 2 shows the inertial component of the lift coefficient. It can be seen that it is considerably less than the dissipative component. The dashed line in Fig. 2 shows levels corresponding to the theoretical values $c_y \simeq 1.0$ and $c_y = \pi/2$ at $\alpha = 45°$. These results were obtained in [6] for steady separated flow about a plate ($\omega = 0$) with a fully formed wake and with a zero wake displacement thickness, respectively.

As when we compared the functions $c_{xD} = f(\omega)$ at $\alpha = 90°$ in Fig. 1, if we compare the theoretical relation with the two empirical relations $c_y = f(\omega)$ at $\alpha = 45°$ from [2, 3], we note a rapid increase in $c_y = f(\omega)$ at $\omega \ll 1$ and a slow increase at $\omega \to 1$. These findings are consistent with the theoretical relation. Arrows 1 and 2 in Fig. 2 show the hysteresis of the lift coefficient when plate rotation is stopped at the angle of attack $\alpha = 45°$ with $\omega = 0.5$ (arrow 1 corresponds to the decrease in the lift coefficient due to disappearance of the unsteady inertial and dissipative components; arrow 2 corresponds to the further reduction in the lift coefficient due to the formation of a full-sized vortex wake behind the plate).

4. After establishing that the present model of deep dynamic stall can explain experimental data on the aerodynamic loads on a wing with $\dot{\alpha}$ = const and large angles of attack, it is interesting to explore the model's ability to describe the dynamic hysteresis of the aerodynamic characteristics for moderate angles of attack and a periodic change in this angle in accordance with the law [1]

$$\alpha = \alpha_0 + \alpha_1 \sin \dot{\alpha} t. \tag{4.1}$$

Figure 3 reproduces experimental relations $c_y = f(\alpha)$ from [1] and [5] for an NACA 0012 wing (at Re of an order of magnitude exceeding those attained in [2, 3]) experiencing a change in the angle of attack in accordance with law (4.1) ($\alpha_0 = 10°$, $\alpha_1 = 10°$). The axis of rotation in this case is located 1/4 of the chord from the leading edge. The results shown are for $\omega = 0.1$ [1] and $\omega = 0.15$ [5]. The flow regime in the tests (with separation of jets from the leading and trailing edges) corresponds to the conditions under which the model is valid, even though the maximum angle of attack is only 20°. To obtain a theoretical estimate of the amplitude of the hysteresis loop of the lift coefficient, we replace the sinusoidal relation in the range $0 < \alpha < 20°$ with a relation consisting of two sections: a linear section (ω = const); a section corresponding to the steady-state flow regime ($\omega = 0$). Then using a method similar to that employed above, we can determine the inertial and dissipative components of c_x and then find c_y.

In Fig. 3, the theoretical relations $c_y = f(\alpha)$ for $\omega = 0.1$ and $\omega = 0.15$ are shown in the range $15° < \alpha < 20°$. Arrow 1 characterizes the increase in c_y with the transition from $\alpha = 15°$ to $\alpha = 20°$ when ω = const, while arrows 2 and 3 show the decrease in c_y accompanying the disappearance of the unsteady inertial component and total dissipative component of the lift coefficient on the section $\omega = 0$. Arrow 4, originating from point B corresponding to steady separated flow about a plate with a fully formed vortex wake behind it, passes below the dashed line corresponding to the same situation and obtained in [6]. However, the relations used in the present study are inadequate for determining the extent to which the lower branch of the hysteresis loop descends below the dashed line. It is necessary to have relations for separated flow about a plate in the presence of a source modeling the wake and the delay in its disappearance with movement along the lower branch of the hysteresis loop (i.e., with a reduction in the angle of attack). Nevertheless, it is clear that the data presented in Fig. 3 is sufficient to demonstrate the quantitative reliability of the model for describing hysteresis associated with deep dyanamic stall when there is a periodic change in the angle of attack within the range of moderate values of this angle (the deviations from the experimental data do not exceed the deviations of the numerical solution of the Navier—Stokes equations from [5]).

In conclusion, we note that at $\omega \ll 1$ and $\dot{\alpha}$ = const (i.e., under conditions characteristic of the problem of "ultra-maneuverability") the main physical reason for the increase in aerodynamic loads is a transition from separated flow with a fully formed wake to wakeless separated flow (to the cases first reported in [6]); all of the other transient effects described here can be ignored due to their small magnitude.

LITERATURE CITED

1. W. J. McCroskey and S. L. Pucci, "Viscous-inviscid interaction on oscillating airfoils in subsonic flow," AIAA J., 20, No. 2 (1982).
2. M. S. Francis and J. E. Keese, "Airfoil dynamic stall performance with large-amplitude motions," AIAA J., 23, No. 11 (1985).
3. G. M. Graham and J. H. Strickland, "An experimental investigation of an airfoil pitching at moderate to high rates to large angles of attack," AIAA Pap., No. 0008, New York (1986).
4. E. J. Jumper, S. J. Shreck, and R. L. Dimmick, "Lift-curve characteristics for airfoil pitching on constant rate," AIAA Pap., No. 0117 (1986).
5. W. Johnson, "Current state of theoretical studies in the aerodynamics of a main rotor," Aérokosmich. Tekh., No. 5 (1987).
6. G. I. Taganov, "Model of the origination of circulation near a wing of infinite span with a sharp trailing edge at large Reynolds numbers," Preprint No. 5, Division of Mechanics of Heterogeneous Media of the Academy of Sciences of the USSR, Moscow (1980).
7. G. I. Taganov, "Substantiation of the relation $\Pi = \rho U_\infty^2 \delta_{2\infty}^{**}$, used in a model of the origination of circulation near a wing of infinite span with a sharp trailing edge," Uch. Zap. TsAGI, 17, No. 5 (1986).
8. G. I. Taganov, "Second dissipative layer and wake in a viscous flow about a body," Uch. Zap. TsAGI, 1, No. 6 (1970).
9. V. S. Sadovskii, "Non-circulating flow of an ideal fluid about a rotating plate," Zh. Prikl. Mekh. Tekh. Fiz., No. 3 (1990).

FIELD OF HORIZONTAL VELOCITIES CREATED BY A MOVING SOURCE OF PERTURBATIONS IN A STRATIFIED FLUID

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A linear formulation is used in the present study to examine a three-dimensional problem concerning determination of the field of horizontal velocities $\mathbf{u}(x, y, z)$ created by a point source moving uniformly and rectilinearly in an inviscid, incompressible, vertically stratified fluid. Formulas representing the exact solution of the problem are obtained in the form of single integrals. In contrast to the solution obtained in [1] for the vertical component of velocity, the expressions obtained here for $\mathbf{u}$ contain nonwave terms which ensure that the series converge. Complete asymptotic expansions of $\mathbf{u}$ are constructed for $x^2 + y^2 \to \infty$ and it is shown that they converge when the contributions of the individual modes are summed. An example of calculation of the components of $\mathbf{u}$ in the nearby region is presented for a homogeneous fluid and a uniformly stratified fluid. It is shown that the singularity normally present in the calculation of wave characteristics in the nearby region is eliminated if the term corresponding to the case of a homogeneous fluid is removed from the solution.

1. Let an inviscid incompressible fluid occupy the region $-\infty < x_1, y < +\infty, -h < z < 0$. The density of the undisturbed fluid $\rho_0(z)$ depends only on the vertical coordinate z and does not decrease with depth. A source of intensity q, located at the depth h_0 reckoned from the position of the undisturbed free surface z = 0, moves at a constant velocity c in the negative direction of x_1 axis. The stationary wave field created by the source is described by the following equations in the coordinate system connected with the source $x = x_1 - ct$

$$\rho_0 D\mathbf{v} = -\nabla p + g\rho, \; D\rho = \rho_0 g^{-1} N^2 w, \; \nabla \mathbf{v} = q\delta(x, y, z + h_0) \tag{1.1}$$

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